Lecture 8

Red-Black Trees: Deletion (contd.), Augmenting Trees

Fixing Violation of Only Property 5

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Case 1: x's sibling w is red.

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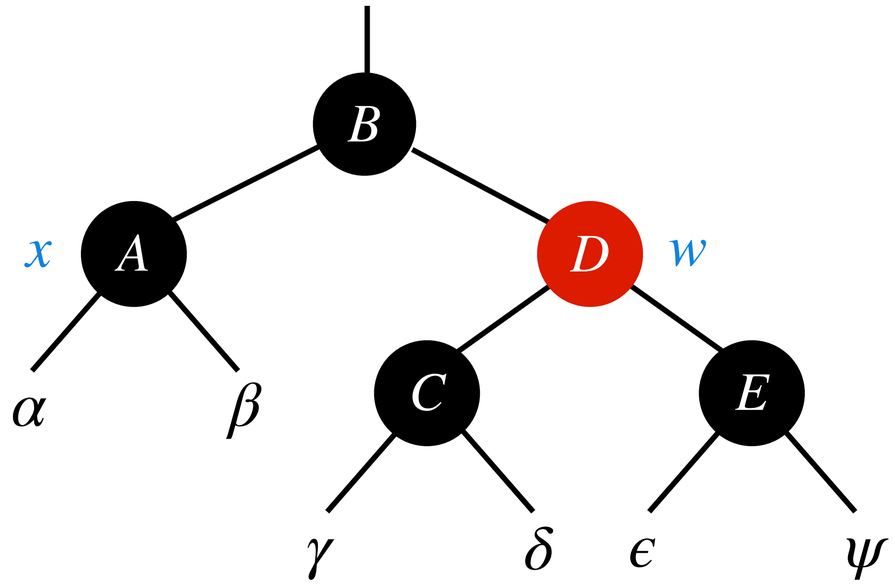

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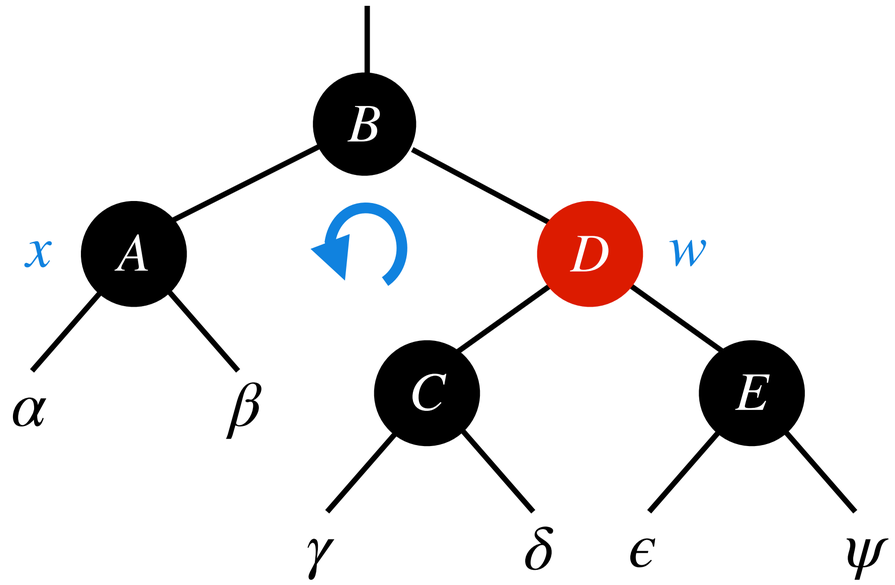

Handling: Switch colours of w and $x\,.\,p$ and perform a left rotation at $x\,.\,p$.

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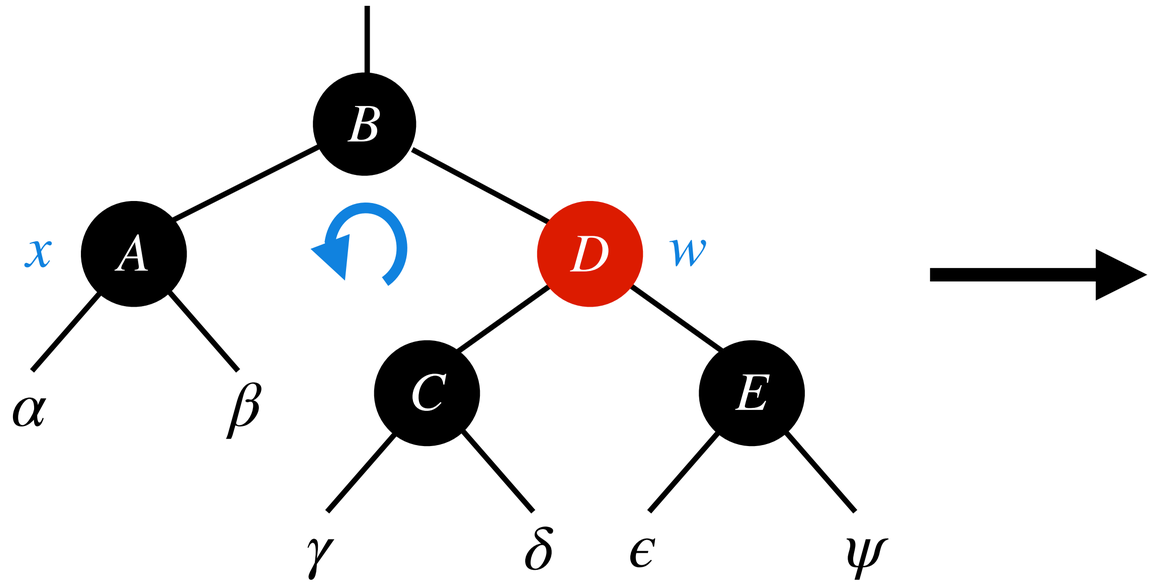

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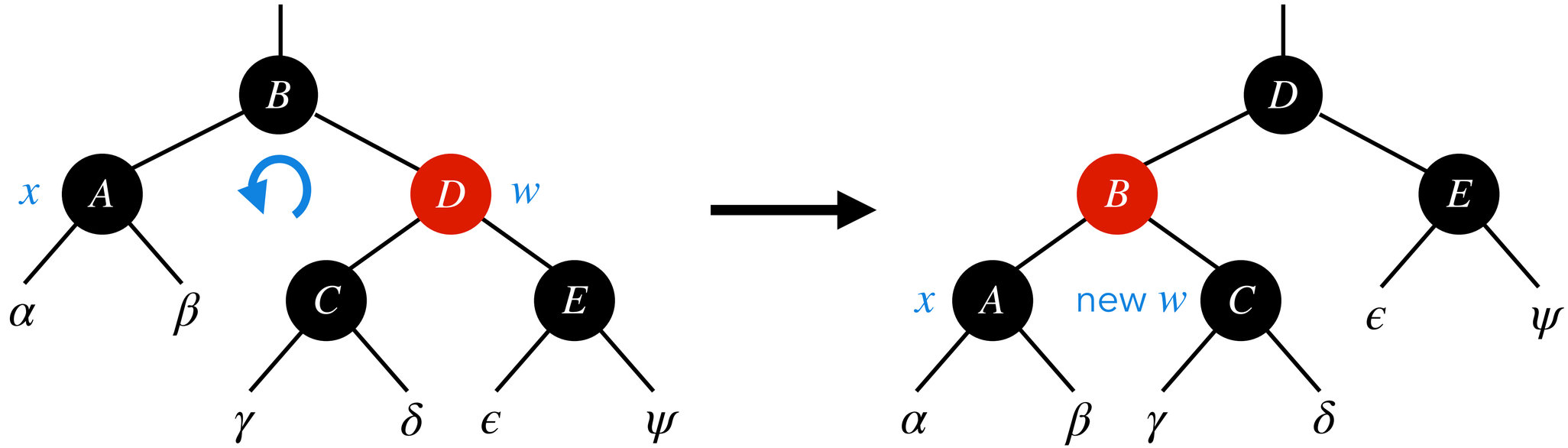

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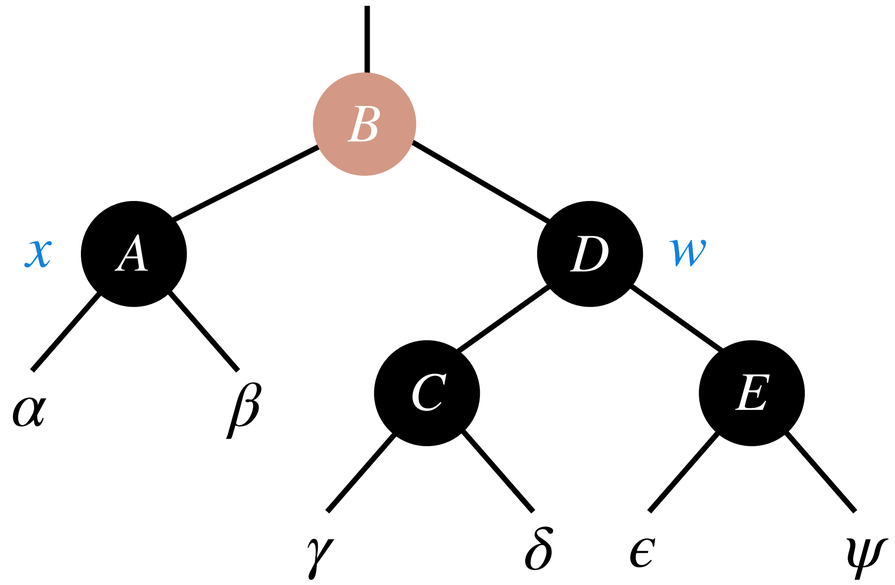

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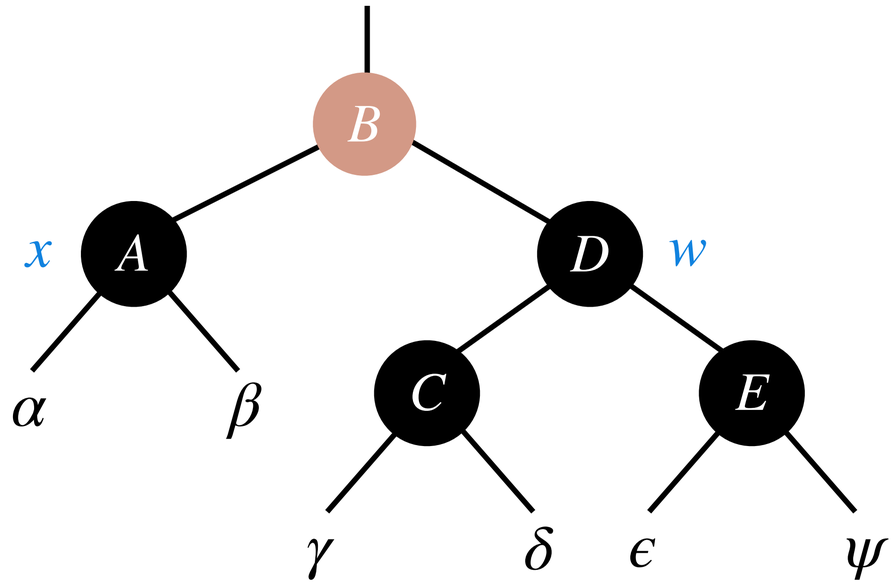

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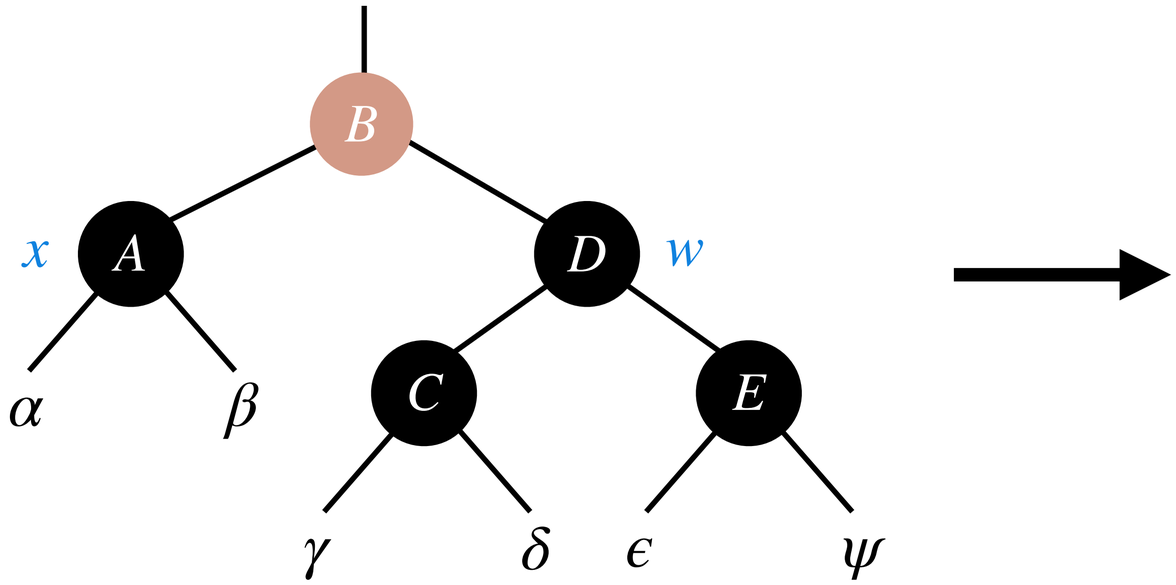

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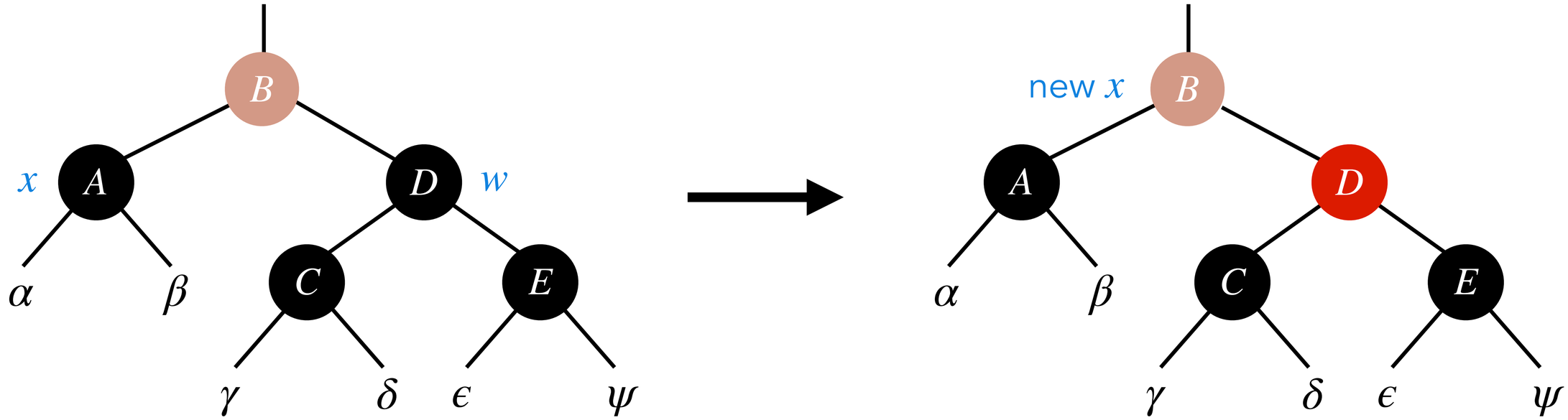

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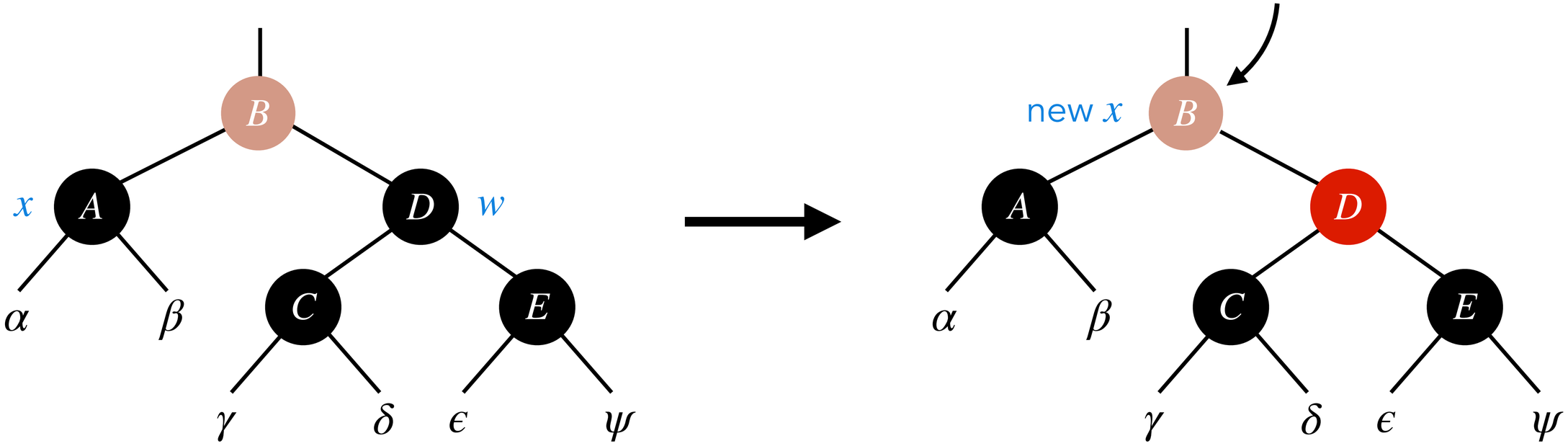
Process can terminate
if B is root or was Red earlier



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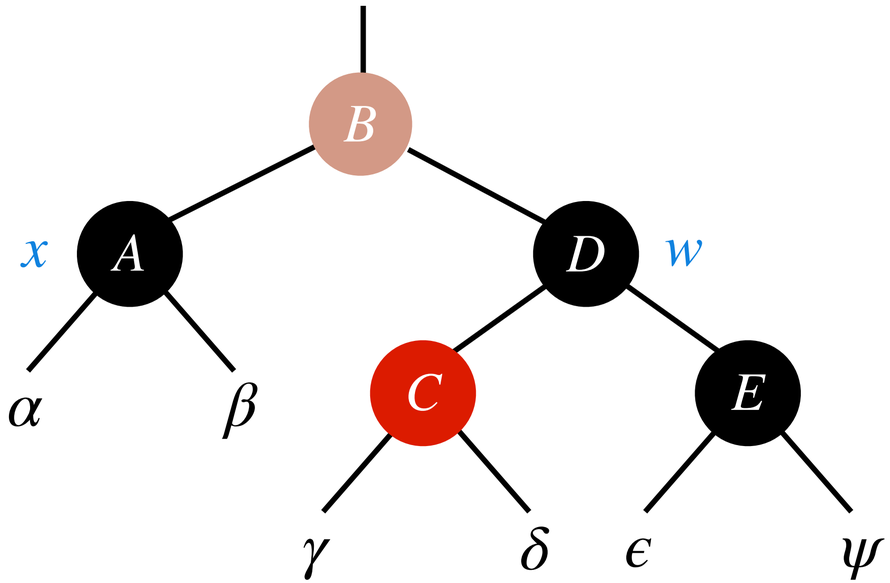

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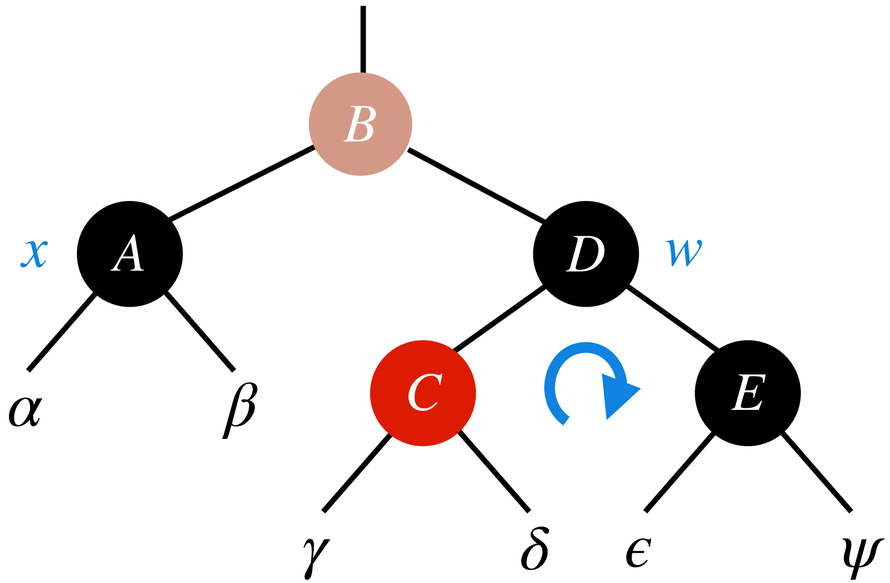

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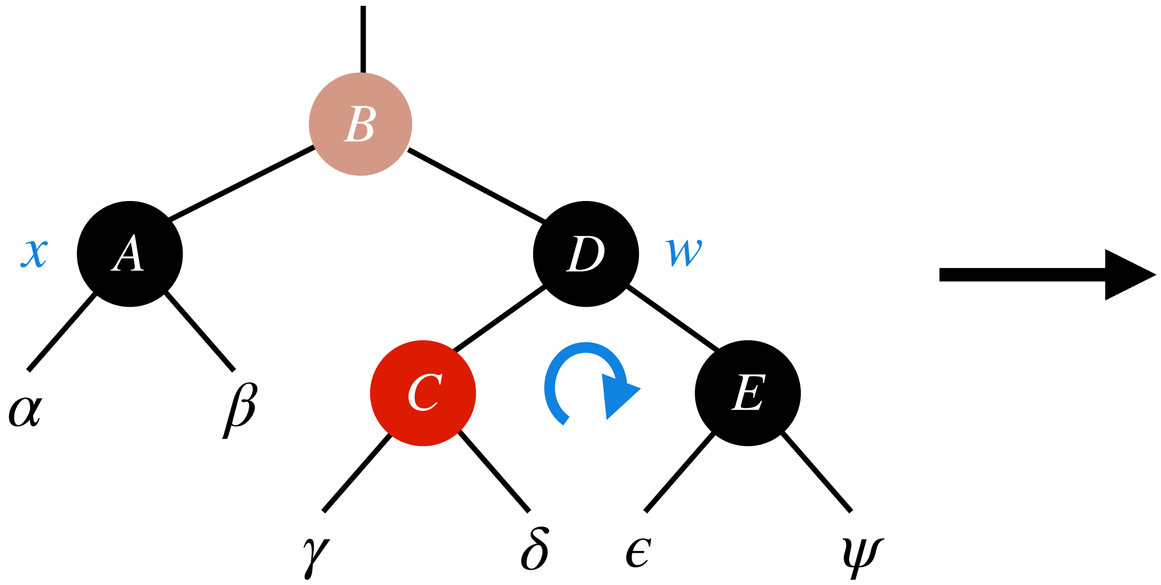

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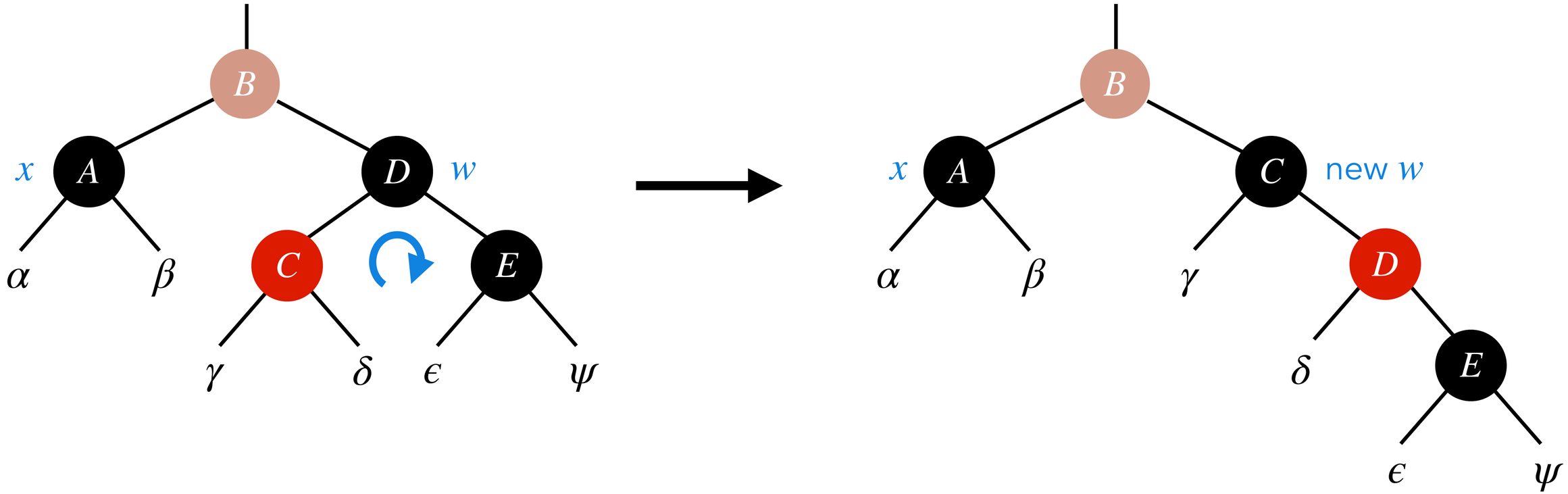

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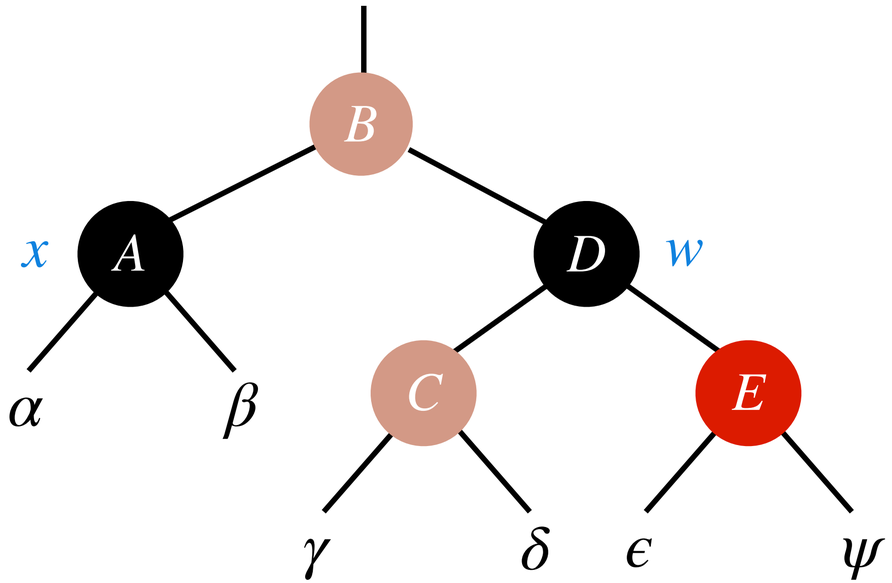

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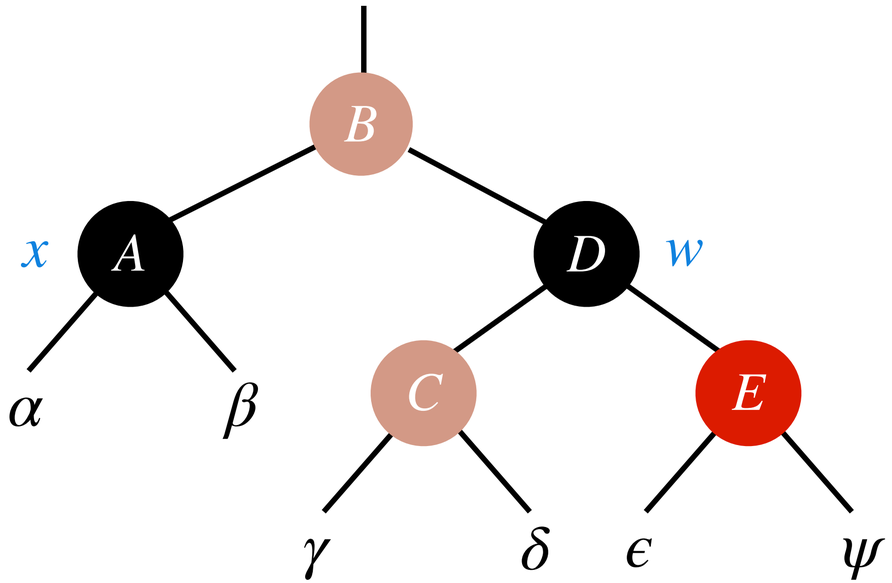

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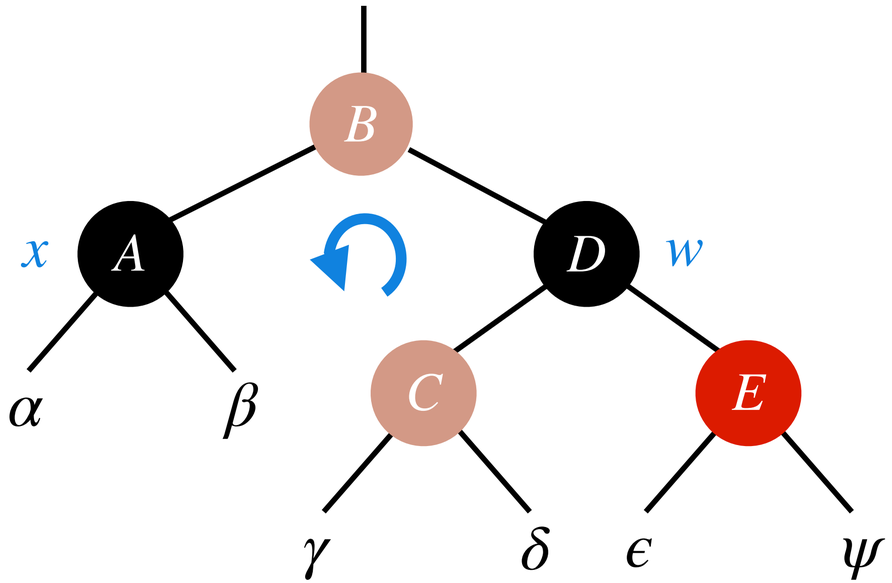

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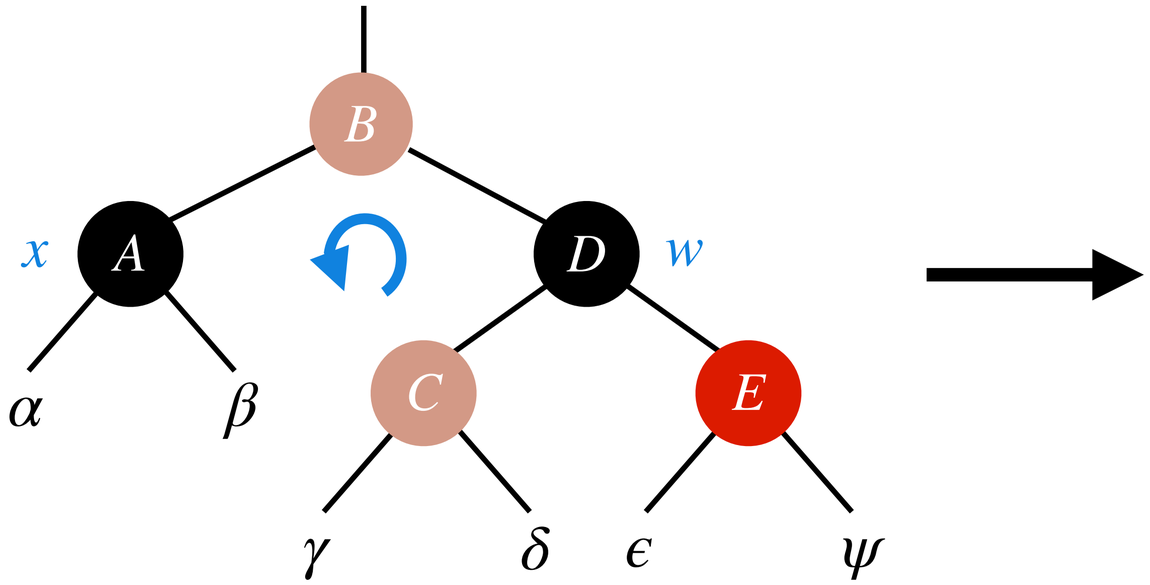

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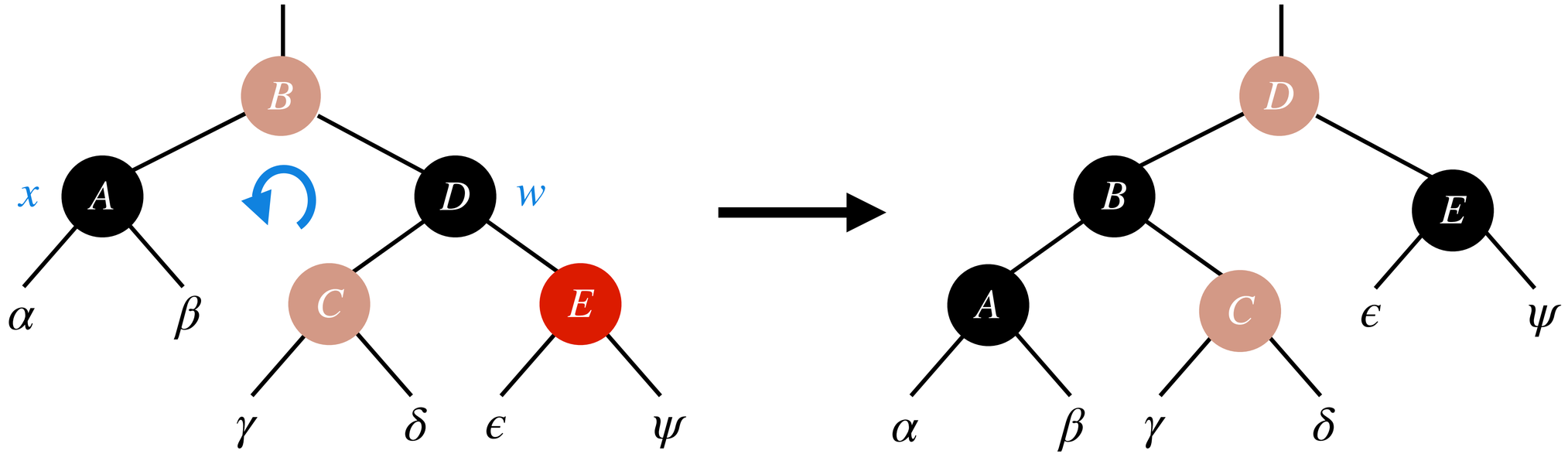

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Ranks of elements 5, 15, and 25, are 1, 4, and 6, respectively.

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No. An easy modification to RB-trees is enough.

A Minor Modification to RB-Tree

Every node also stores the number of internal nodes in the subtree rooted at that node.

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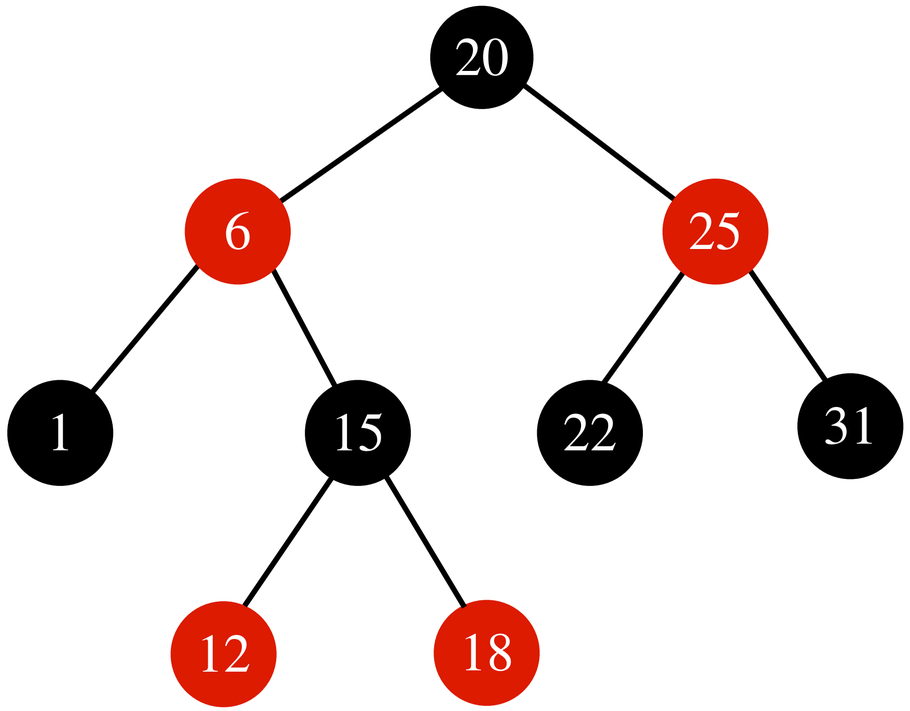

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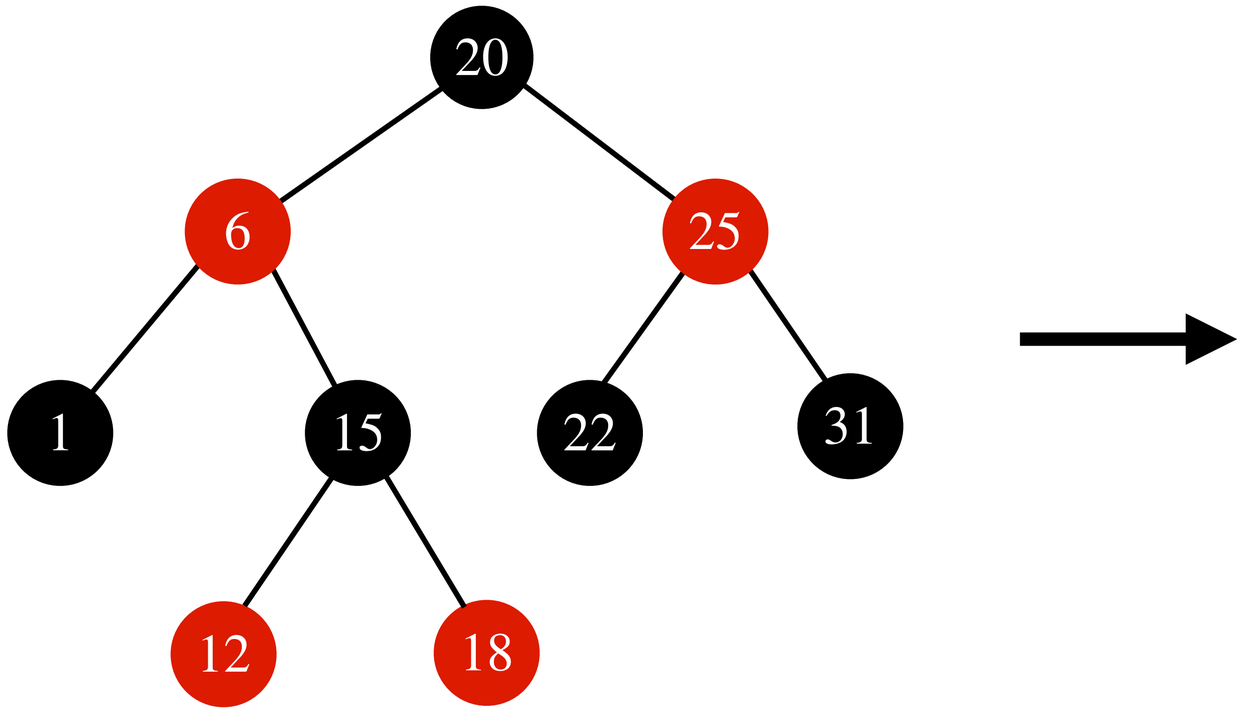

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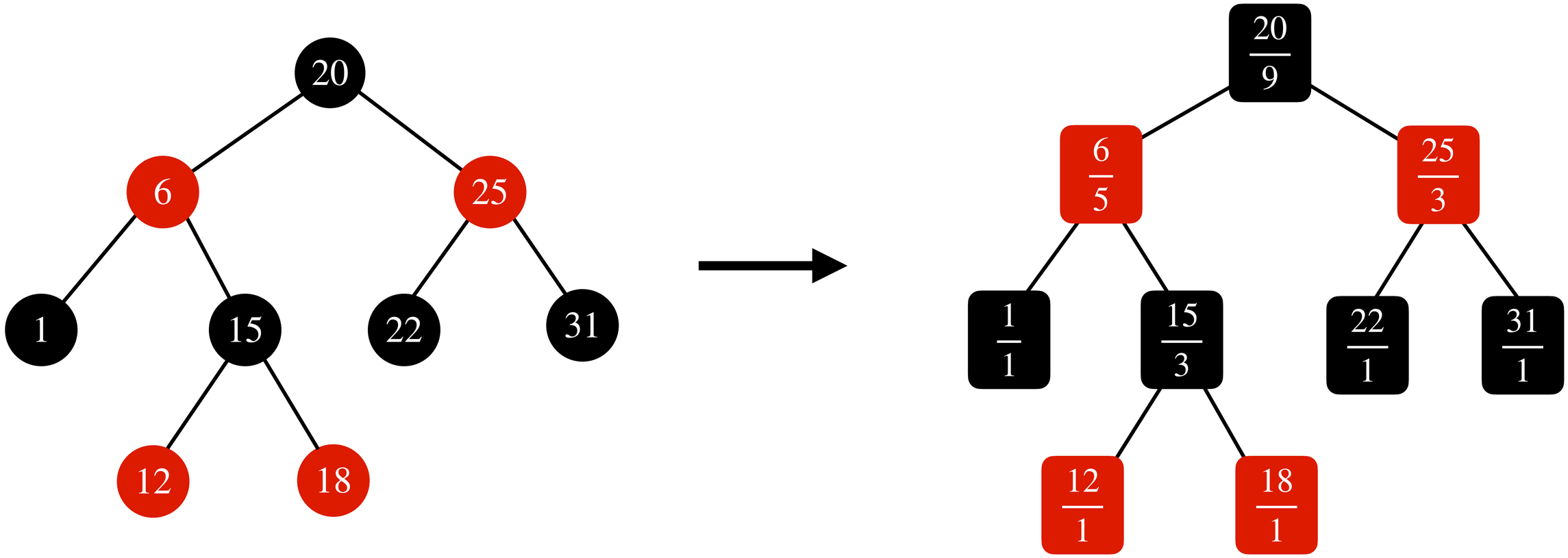

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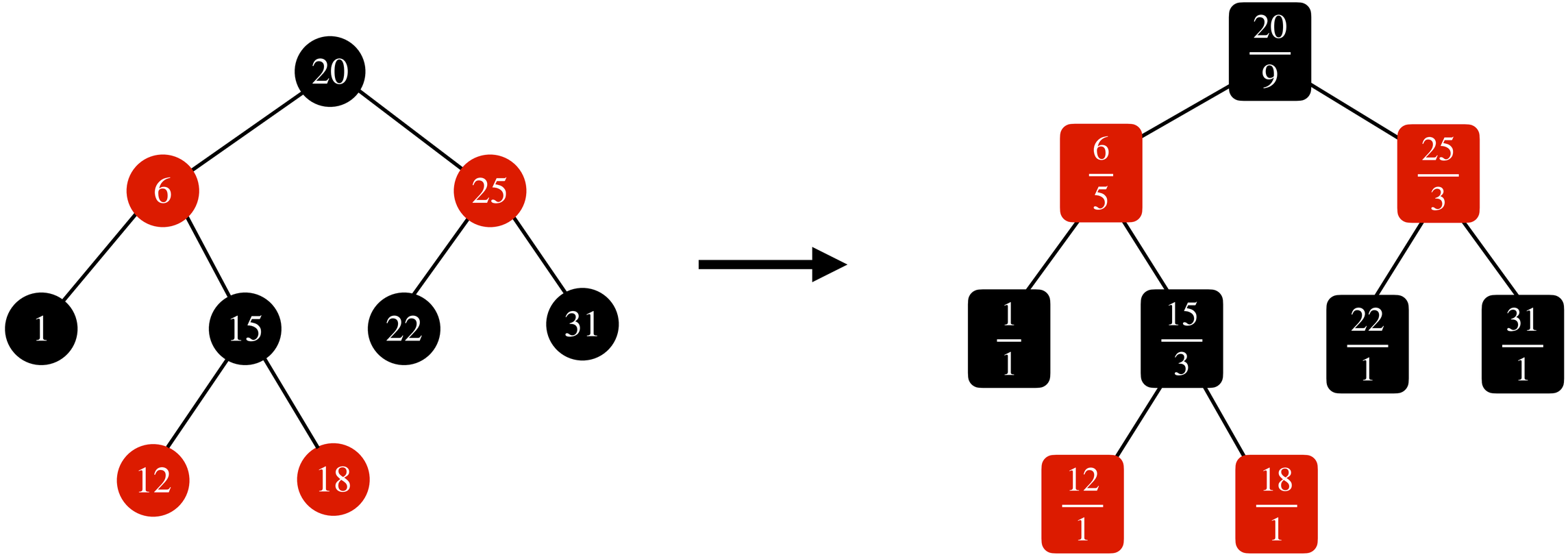
Nodes are represented as $\frac{x}{y}$, where x is the key, and y is number of internal nodes in the subtree



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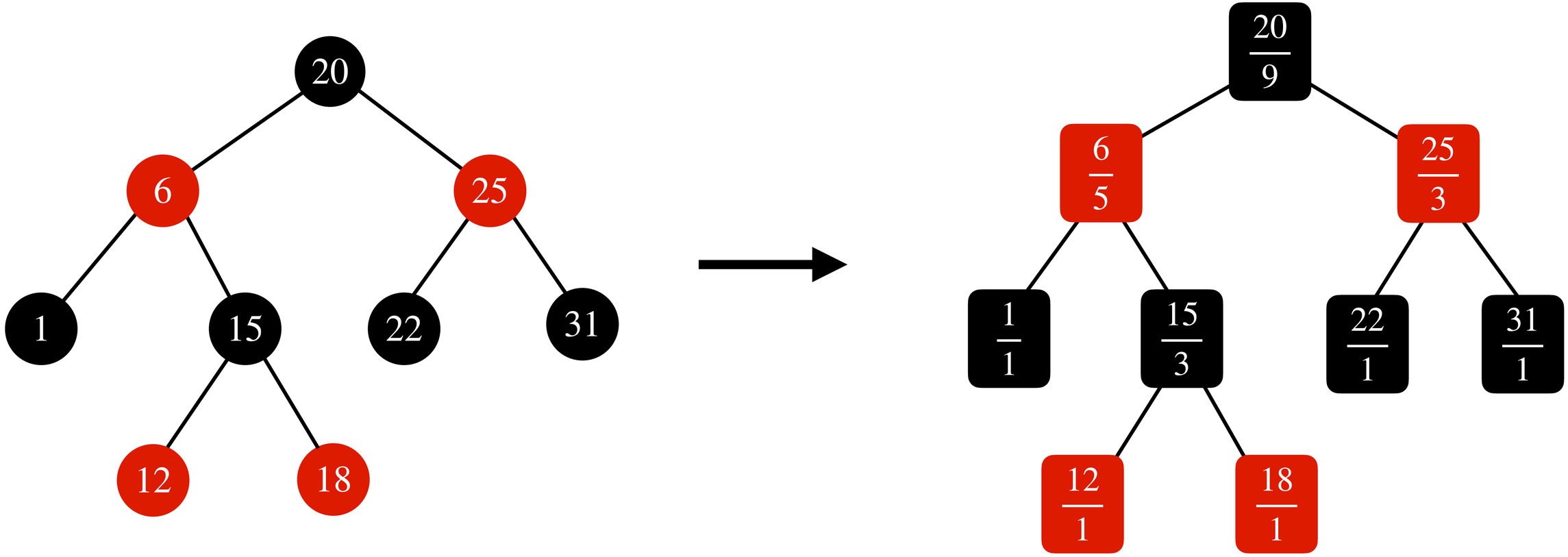

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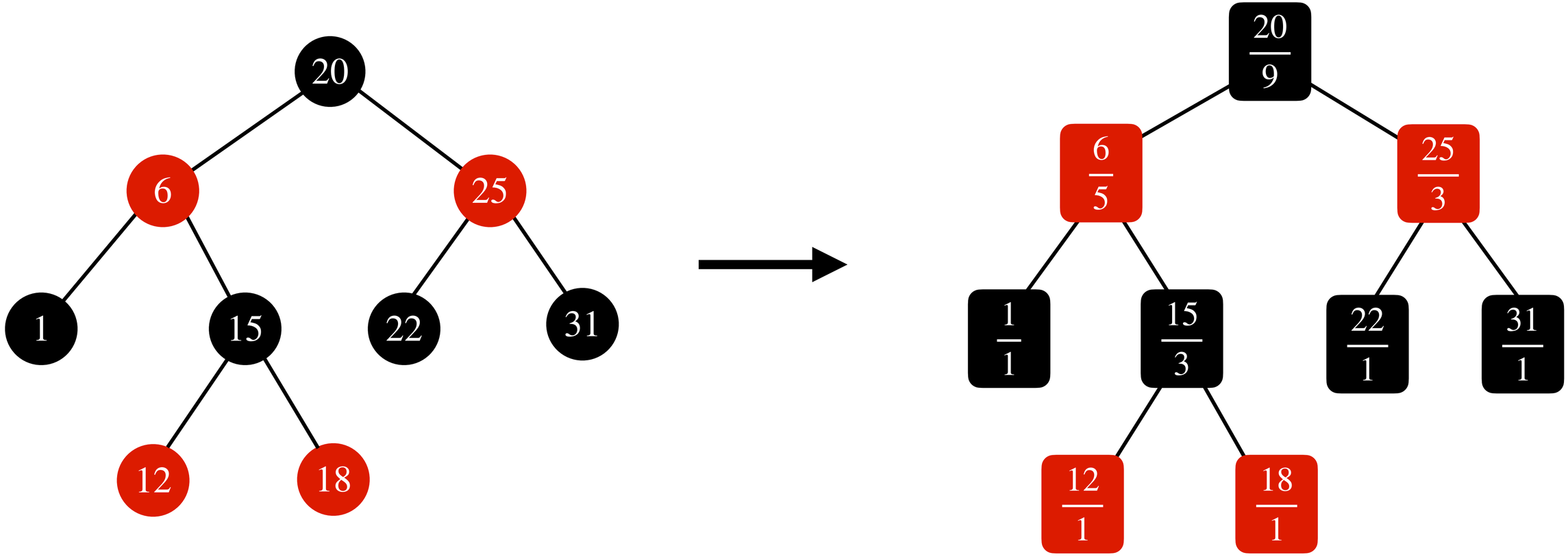

Finding the Element with ith Rank

Recall that rank of an element is its position in the sorted order (w.r.t. keys) of the set.

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Example: Find an element with 15th rank in the below **set** or RB-tree.

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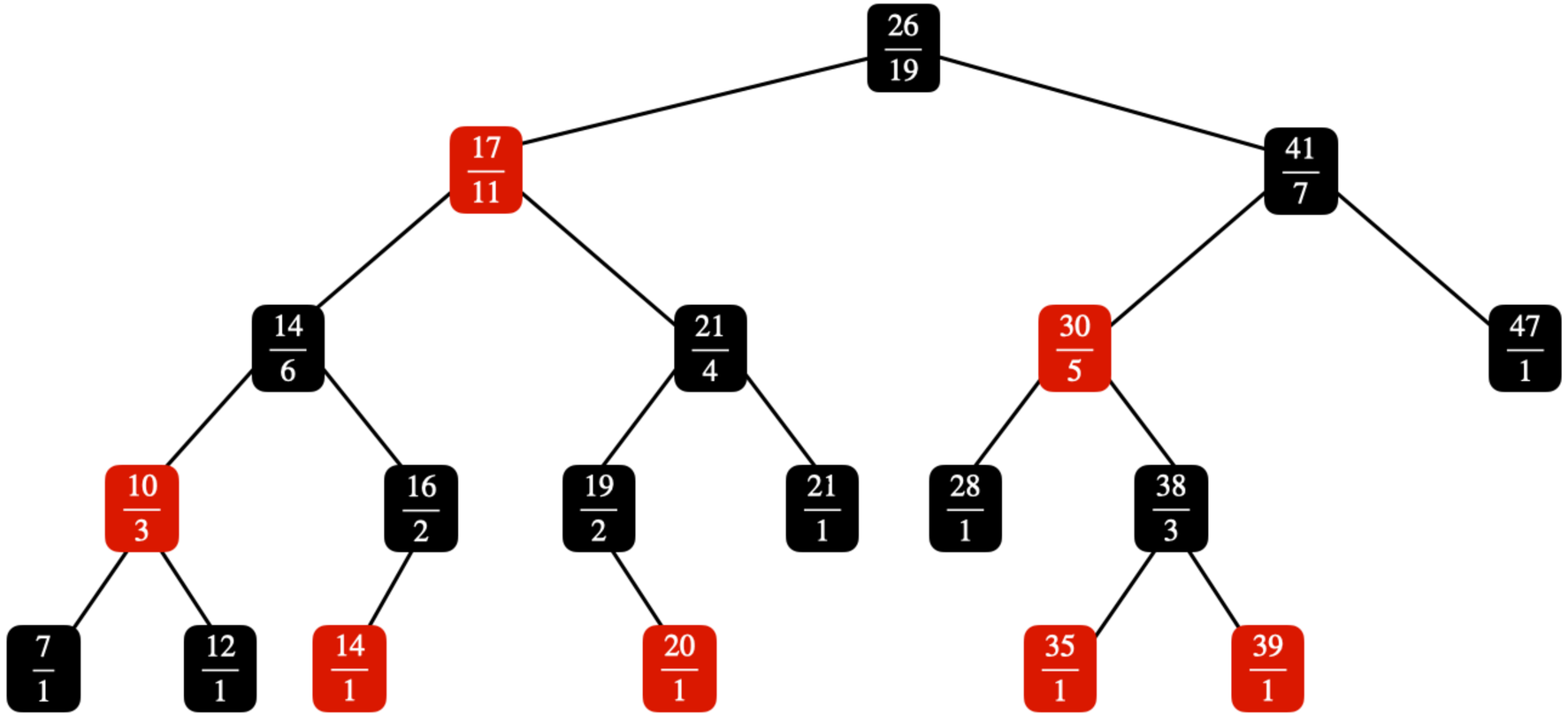

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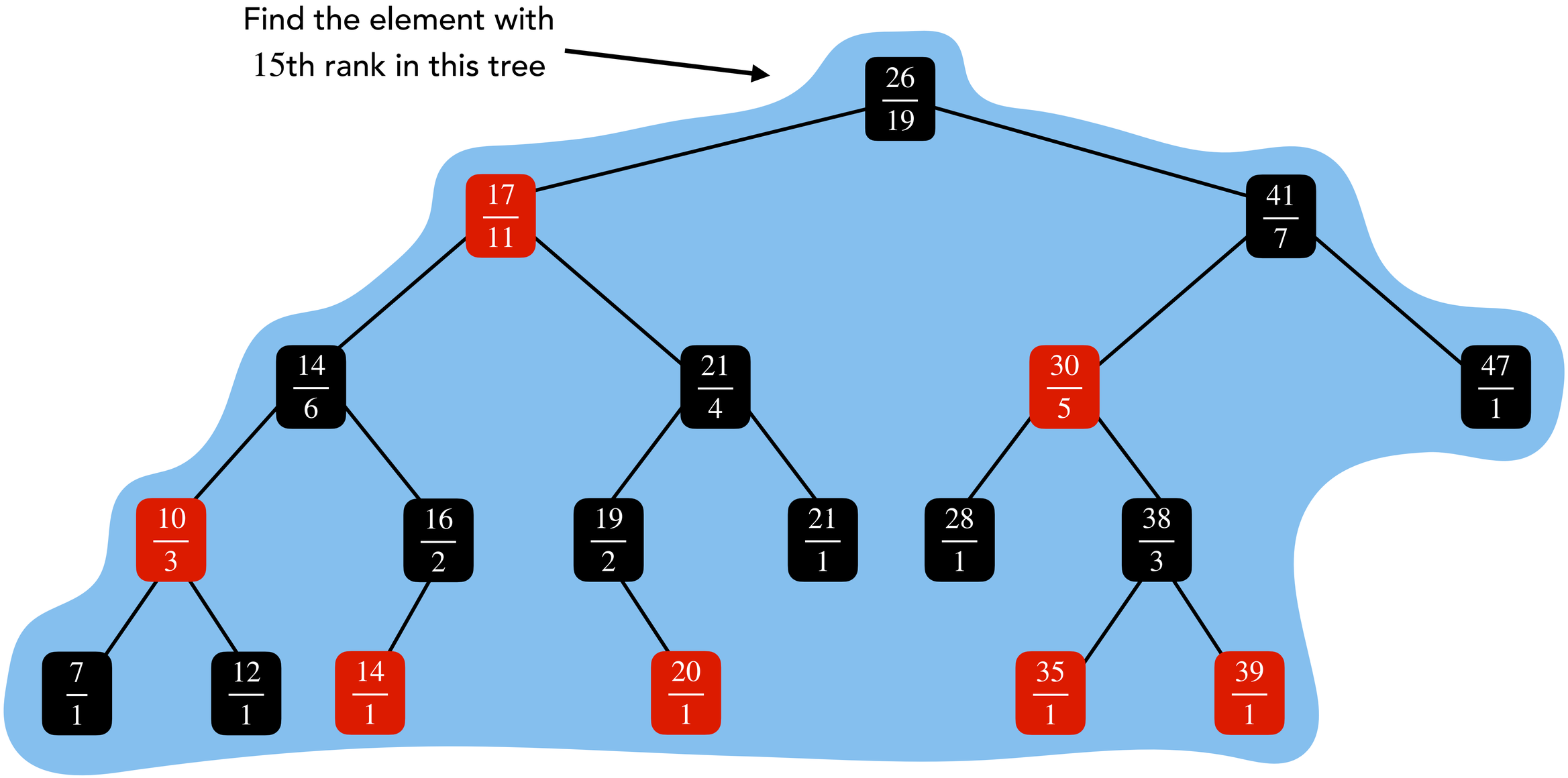
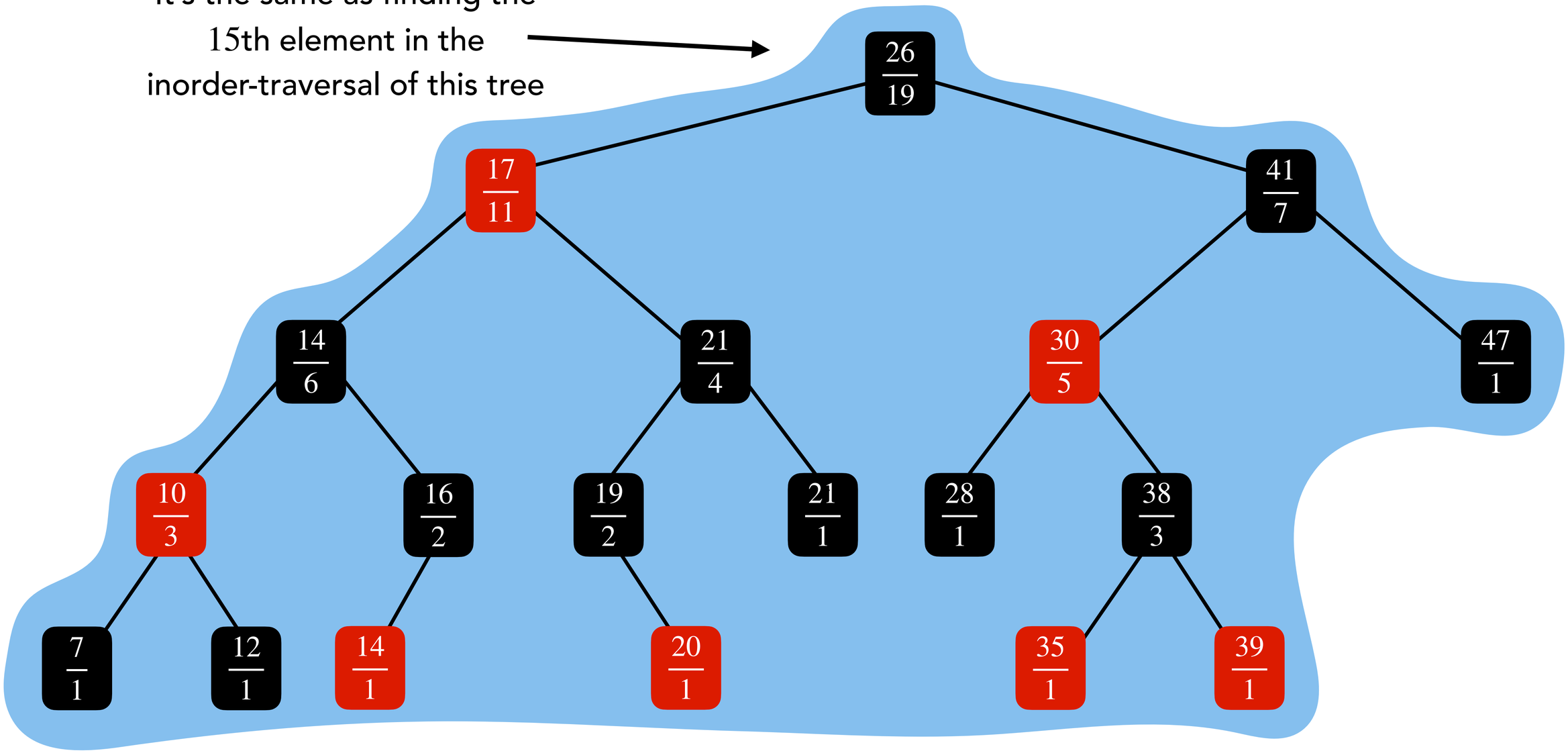


Finding the Element with ith Rank

It's the same as finding the 15th element in the inorder-traversal of this tree



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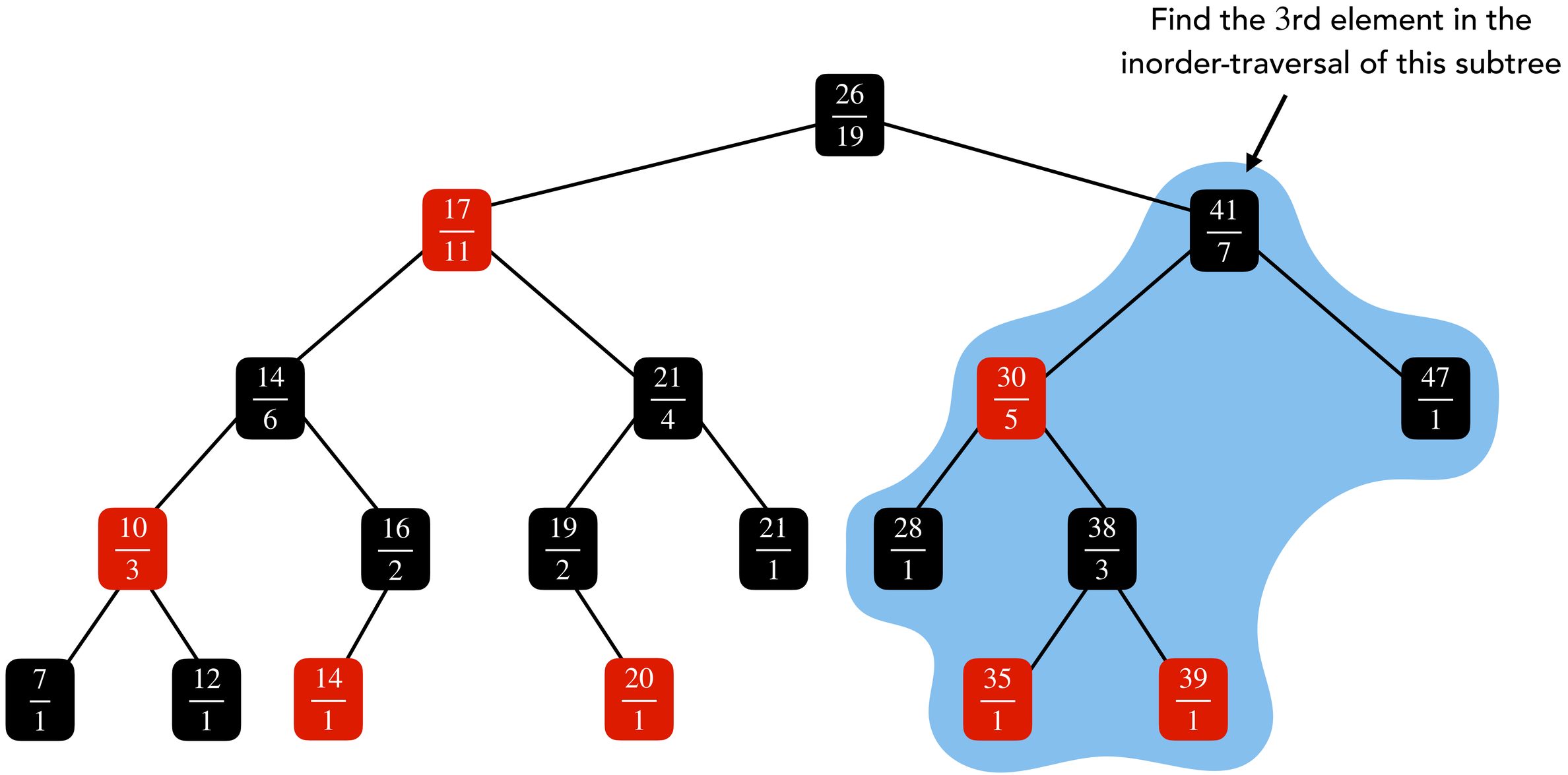

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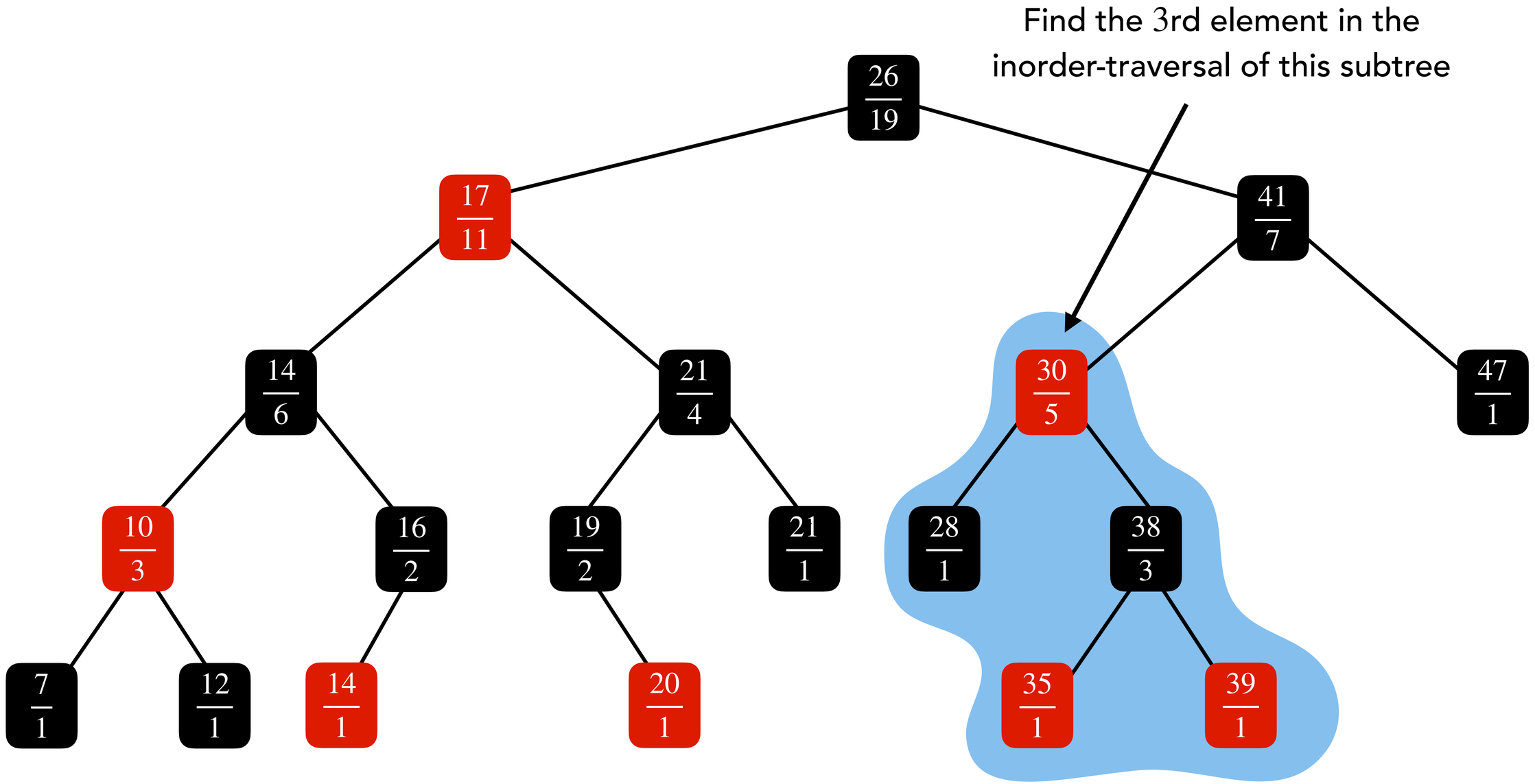

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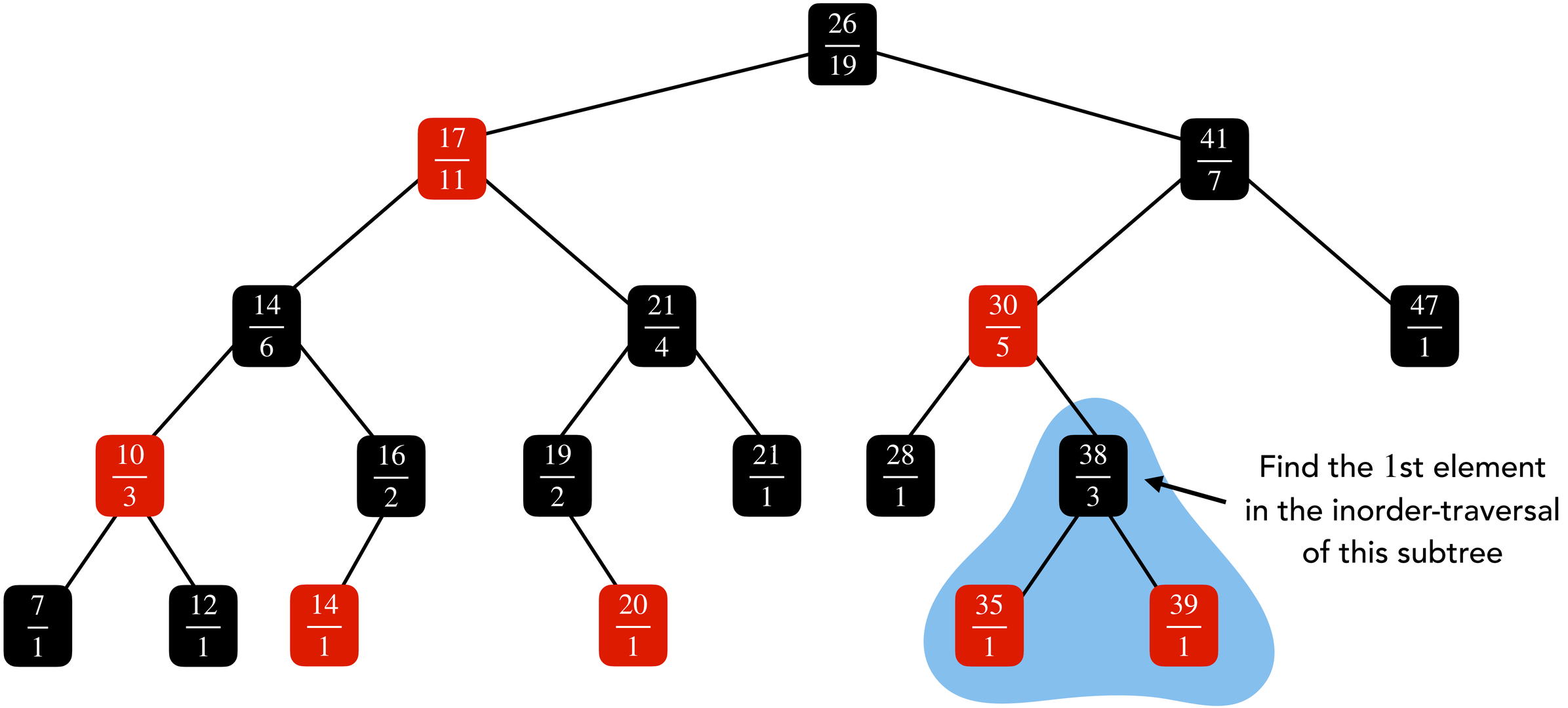

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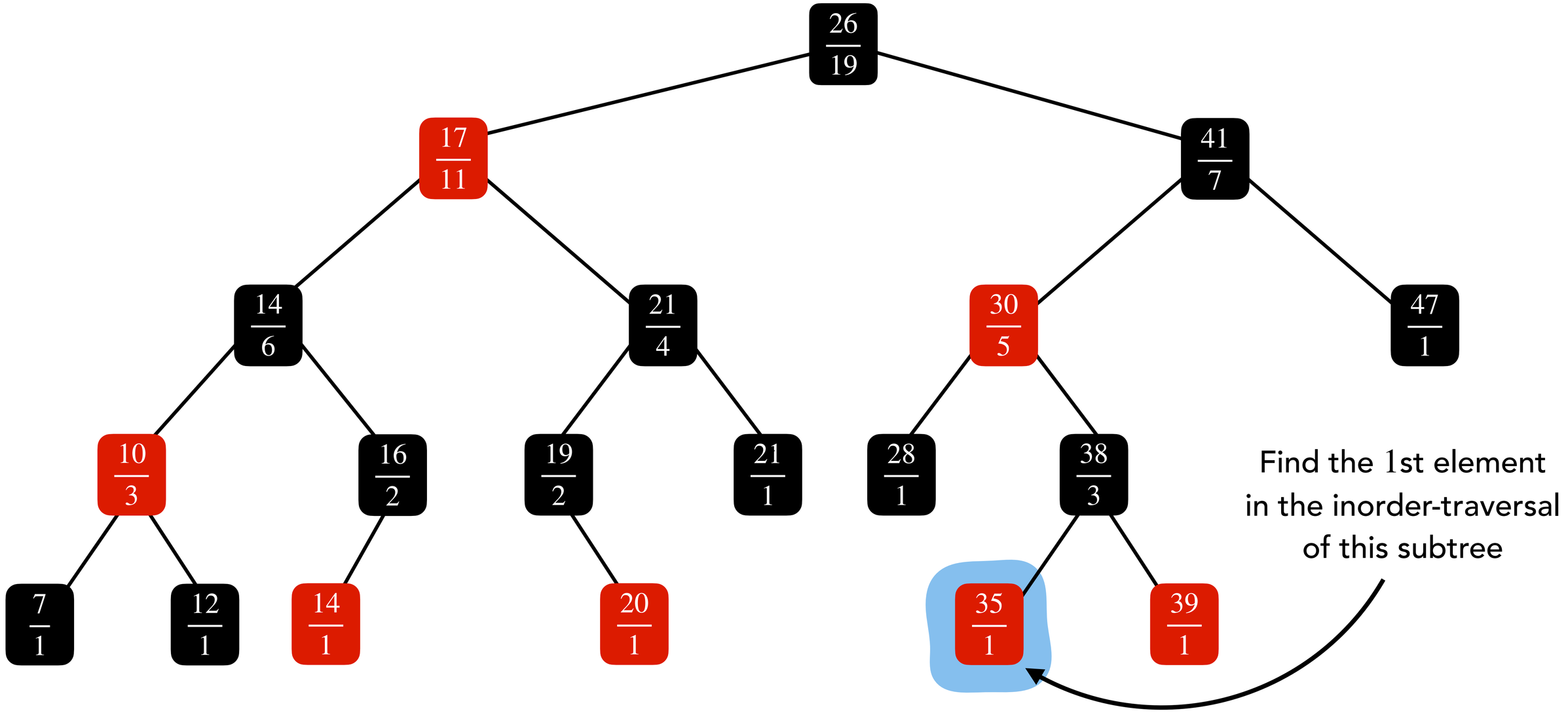

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